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Thunderstorm Electrification

by

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1. Introduction

The previous lecture described, and in part sought to explain, the lightning discharge. This presentation concentrates on the various mechanisms by which electric charge may be generated and separated in sufficient quantity, and at the rate required to supply such discharges.

The lecture is in two parts: Firstly, the properties of thunderstorms, as observed in various field studies of the phenomenon, are discussed; the objective being to identify criteria which an electrification theory should satisfy. Such a list was compiled by Mason (1953,1971) and remains valid now; except that recent studies have allowed some refinement. Secondly, the various contending theories are described and, as far as possible, evaluated. This consists of a discussion of a number of laboratory and numerical modelling experiments which have been carried out, mainly but not exclusively, in the last ten years. This interplay of field study, laboratory experiment and numerical modelling is one of the important features of this field of endeavour. A shortage of good, comprehensive field studies has begun to be corrected in recent years.

2. Some properties of the thunderstorm

In essence, the thunderstorm may be regarded as an electrostatic generator that produces positive and negative electrical charges; the positive charge being concentrated in one region of the cloud and the negative charge becoming concentrated in another. As physical separation of the charges of different polarity proceeds, the electric field and potential difference between the charged regions (or between one of them and the earth or ionosphere) grows until a lightning flash is initiated. At least part of the separated charge is neutralised by the discharge but the field builds up again and the sequence of events is repeated.

A satisfactory theory must explain quantitatively how charge is generated, and separated at a sufficient rate to account for its dissipation in lightning flashes. It must account for the observed distribution of positive and negative charges and the observed changes of electric field produced by the discharges. The theory must also be consistent with the size and life cycle of the storm and its microphysical properties, represented typically by the scale, intensity and duration of the precipitation that accompany it.

2.1 Time and spatial scales

Exploration by aircraft and radar reveals that thunderstorms generally consist of one or more 'cells' that contain strong vertical motions and are the locale of raindrop, and hailstone formation. Each cell grows, reaches maturity and decays. The growth stage lasts for about 10-15

minutes, is characterised by strong updraughts, $\sim$ 10 ms^{-1}, and the initiation of precipitation in the form of rain or hail in the middle troposphere. The onset of the mature stage coincides with the spread of precipitation towards the ground and the appearance of both updraughts and downdraughts, at least in the lower part of the cell. The mature stage lasts for about 30 minutes. Lightning activity occurs predominantly during this phase. Thus any theory must be capable of generating and separating charge sufficient to supply the first lightning stroke within about 10 to 20 minutes of the appearance of precipitation of radar detectable size. This activity should be capable of being sustained for up to 30 minutes.

The cell may be about 1 km across when first detected by radar, but it usually develops rapidly up towards the -20°C isotherm and beyond, and extends to several kilometres in all directions. A feature some 10 km high and 8 km across results.

The base of the thunderstorm is almost invariably warmer than °C, and the cloud top is often cooler than -40°C. In view of earlier discussions in lecture 2, it is to be expected that the intervening cloud and precipitation contains both ice and liquid water forms.

2.2 Charge distributions

The distribution of charge inside a thundercloud was first deduced by Wilson (1920) from the variation of electric field at the ground caused by lightning flashes, as a function of distance from the storm. He concluded that there was a positively charged upper region and negatively charged region lower down, as shown below.

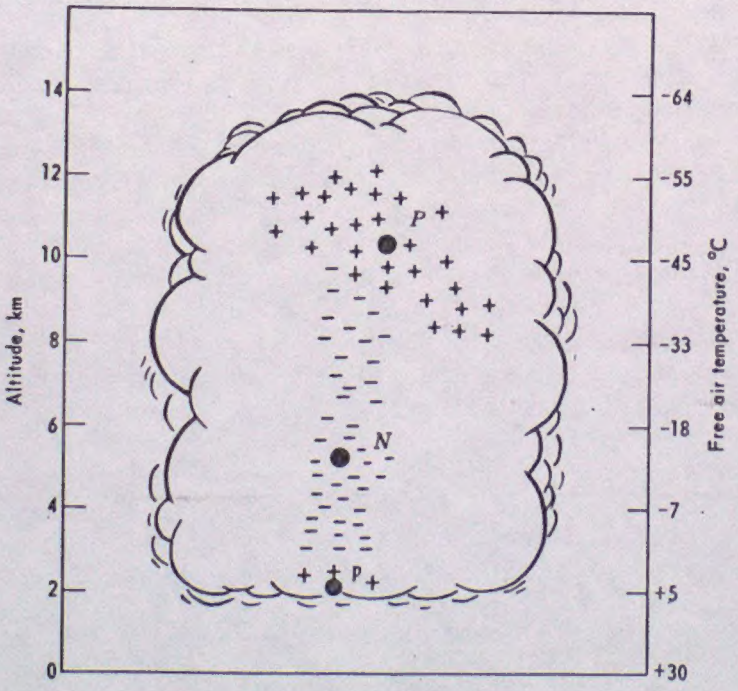


Fig. 1. *Probable distribution of the thundercloud charges, P, N and p for a South African thundercloud according to Malan (1952, 1963). Solid black circles indicate locations of effective point charges, typically P = +40 coul, N = −40 coul, and p = +10 coul, to give observed electric field intensity in the vicinity of the thundercloud.*

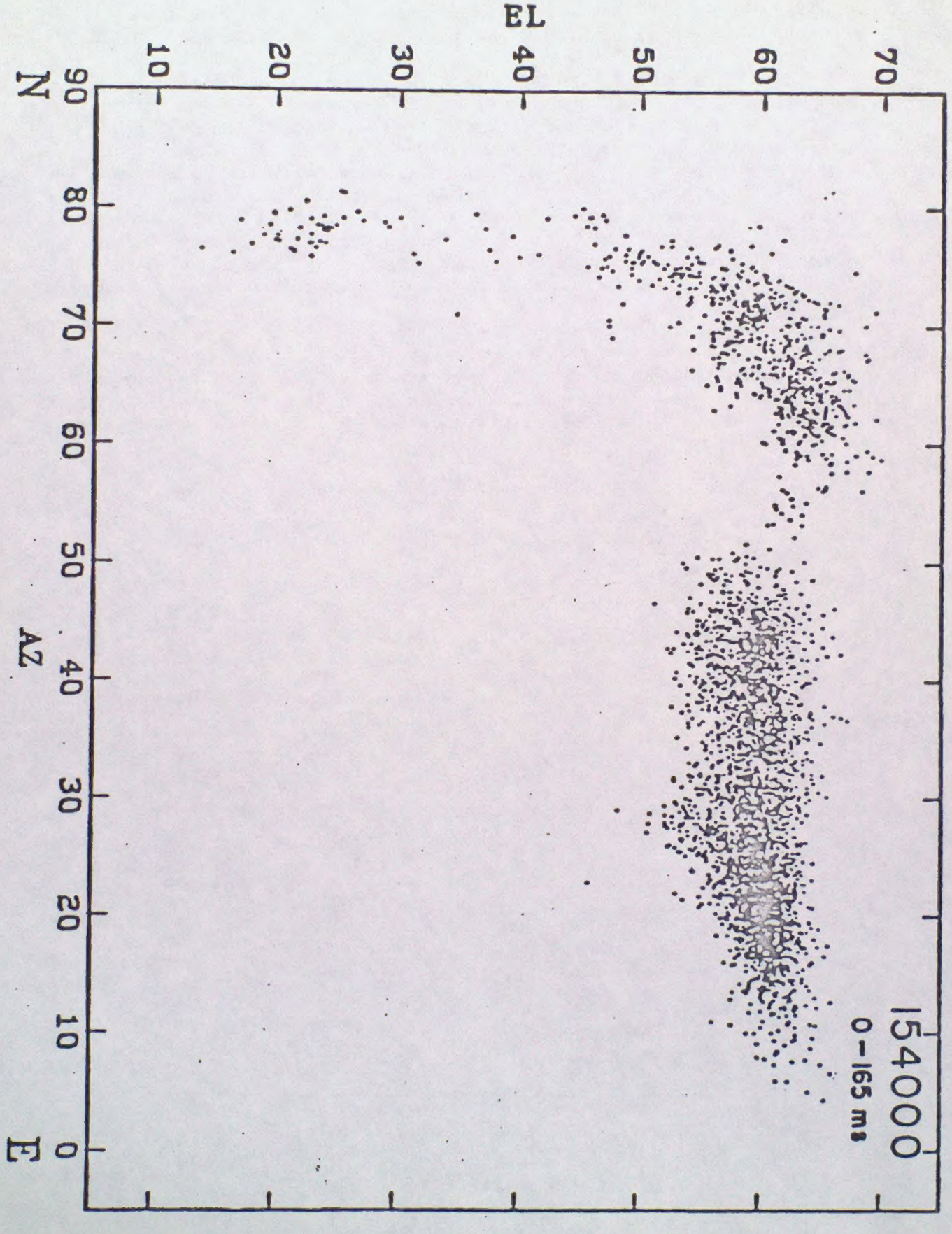


Figure 2 Azimuth, AZ, and elevation, EL, angles in degrees derived from individual radiation sources for the initial portion of a lightning flash 152902 on 29 August 1978, (from Hayenga and Warwick, 1980)

The existence of a pocket of positive charge in the base of some thunderclouds was indicated on some of his records, and has been inferred by Simpson and Robinson (1941) from balloon borne measurements of electric field. More recent measurements referred to below, confirm this general picture as well as providing additional insight.

During the past few years several ground based techniques have been utilised to locate some segments of a lightning flash. Warwick et al (1979) and Hayenga and Warwick (1980) have described a radio interferometric technique for measuring the direction cosines of the centroid of discharges emitting VHF radio signals. The averaging period is approximately 5 μs so that these centroids are likely to define discrete sources. An example of the positional detail provided by this technique is shown in figure 2. The discharges are marked by extensive horizontal development, perhaps the positive streamers discussed earlier, before a more or less vertical discharge to ground is initiated.

Proctor (1980) has obtained similar results in South Africa by multi station reception and timing of VHF noise pulses emitted by discharges. Some of his results are shown in figure 3. All originate in areas of high radar reflectivity between -5 and -16°C.

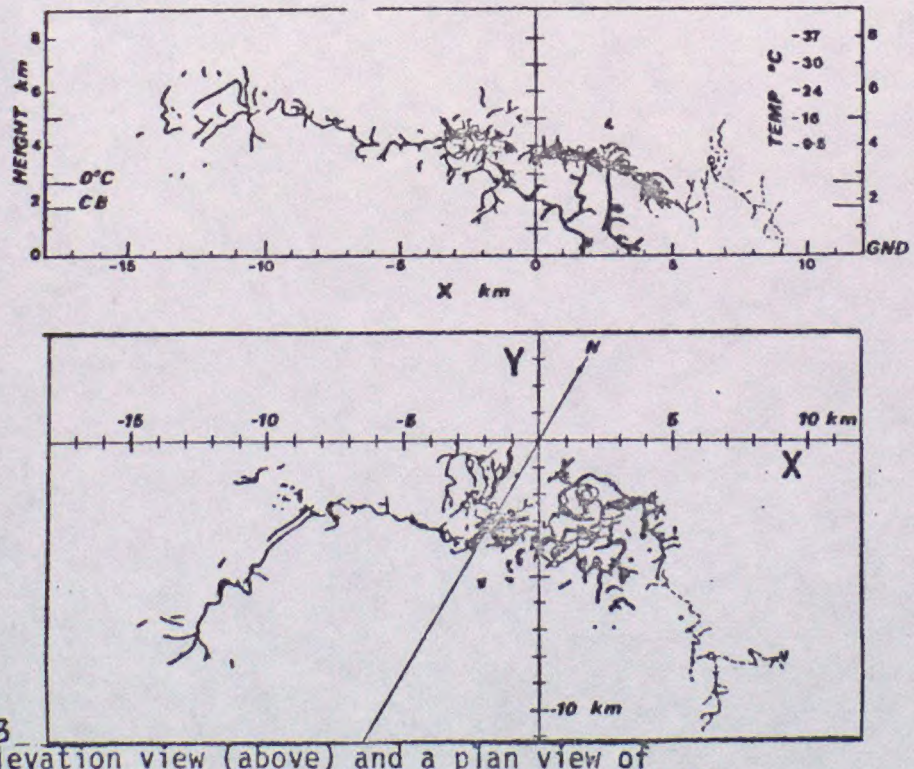


Fig 3 An elevation view (above) and a plan view of seven flashes recorded between 131707 and 131759 on 8th April 1979

Krehbiel, Brook and McCrory (1979) have made multistation measurements of the electric field change induced by lightning in New Mexico storms. A typical set of observations is shown in figure 4.

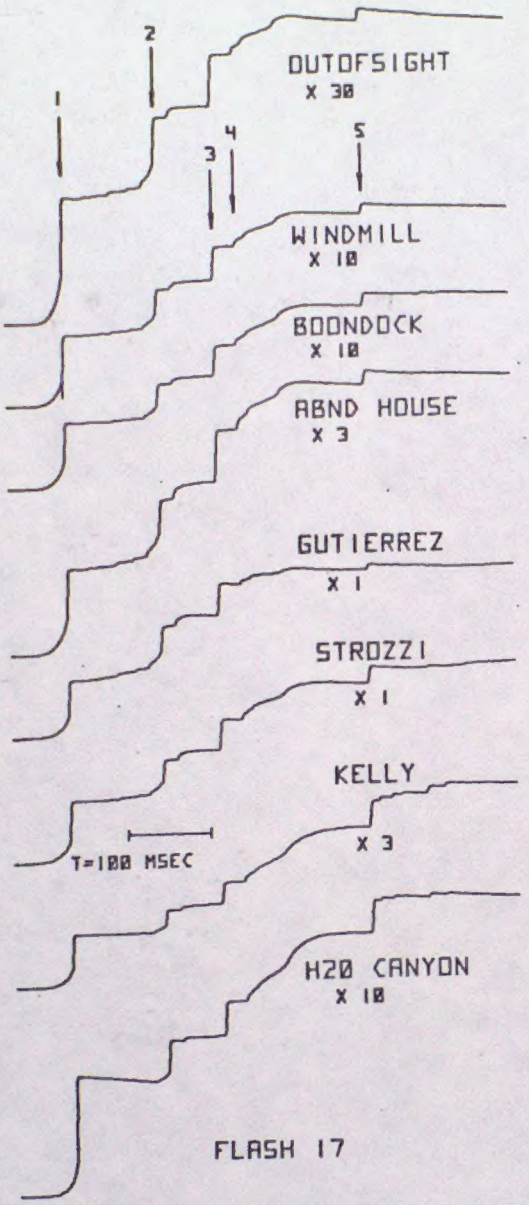


Fig. 4. Electric field change versus time for flash 17, 1252:36 MST. Total field change values range from 22.9 kV/m at the Strozzi station to 1.65 kV/m at the Outofsight station. The negative spike on the Windmill record is an artifice, caused probably by radio interference from the discharge. The small stroke after stroke 2 was not analyzed.

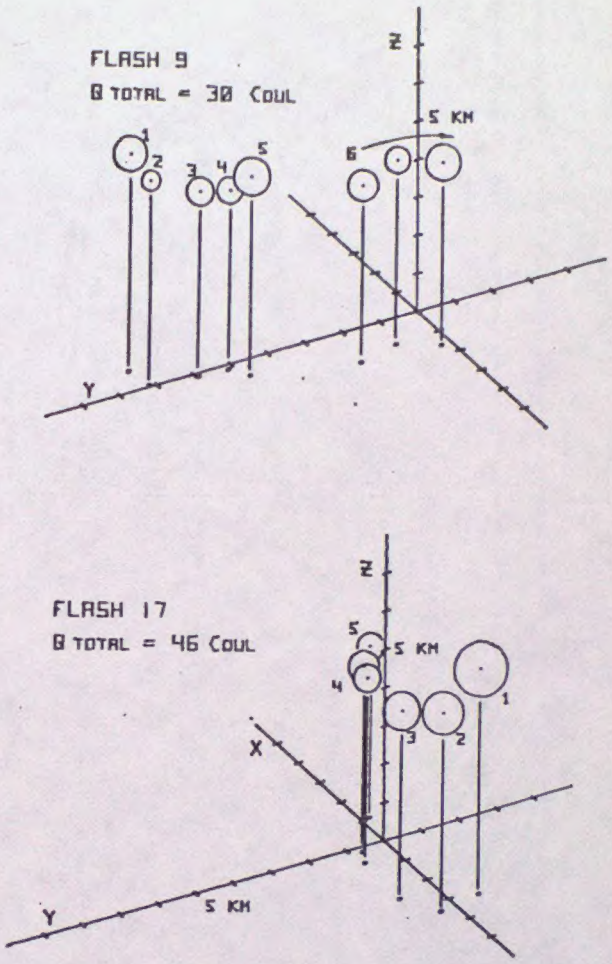


Figure 5 Inferred ground stroke charge for flashes 9, 17. The circles denote the size of spherical volumes which would have contained the individual stroke charges at a uniform density of 20C km^{-3}.

The equivalent charge centres neutralised by the ground flashes have been located from such field change data - see figure 5. The field changes of individual strokes were found to be reasonably consistent with the lowering to ground of a localised, spherically symmetrical negative charge in the cloud. The centres of charge for successive strokes of each flash developed over large horizontal distances within cloud (up to 8 km) at more or less constant elevation between the -9 and the -17°C clear-air-temperature isotherms. Comparison with 3 cm radar measurements of precipitation structure, showed that the discharge

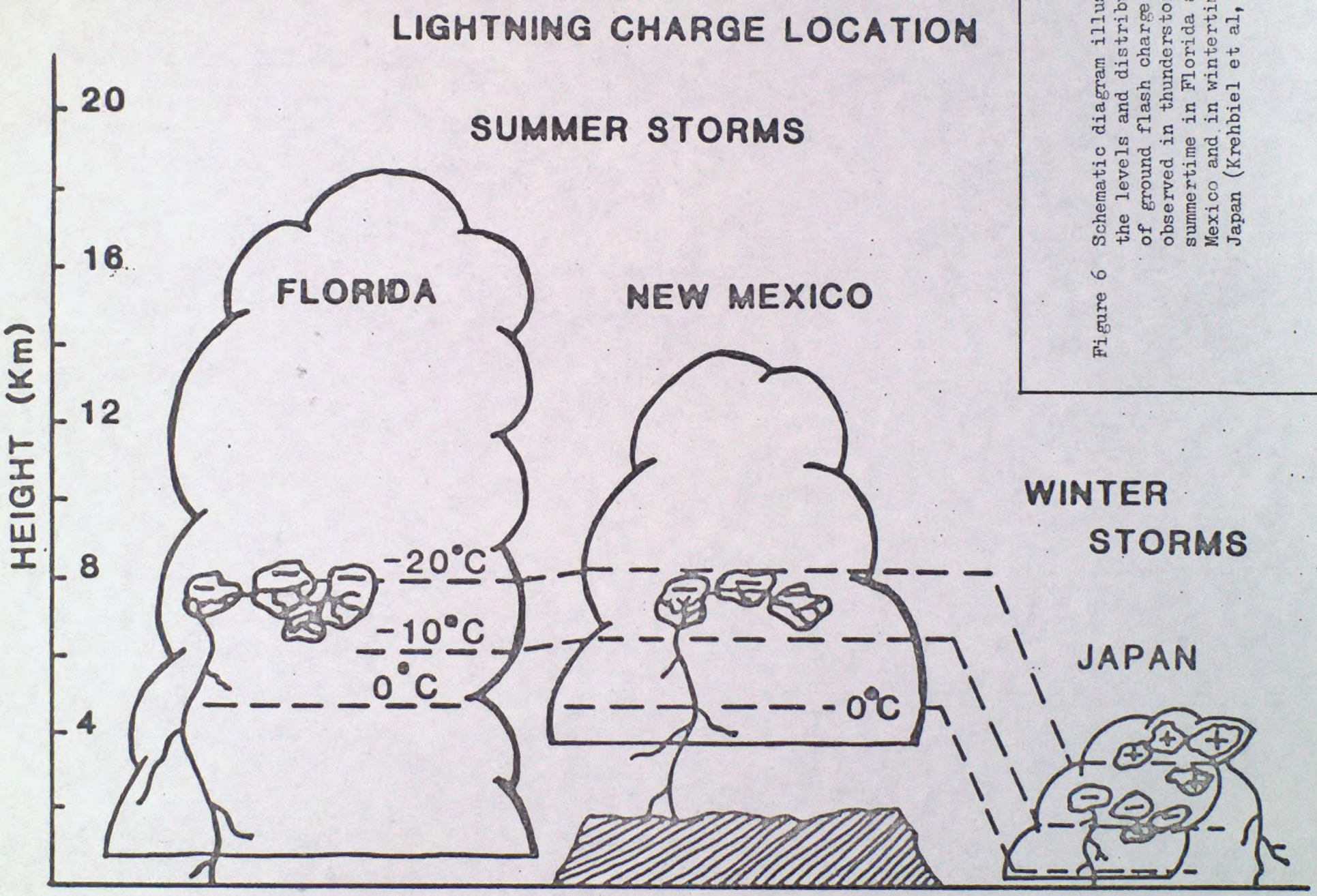


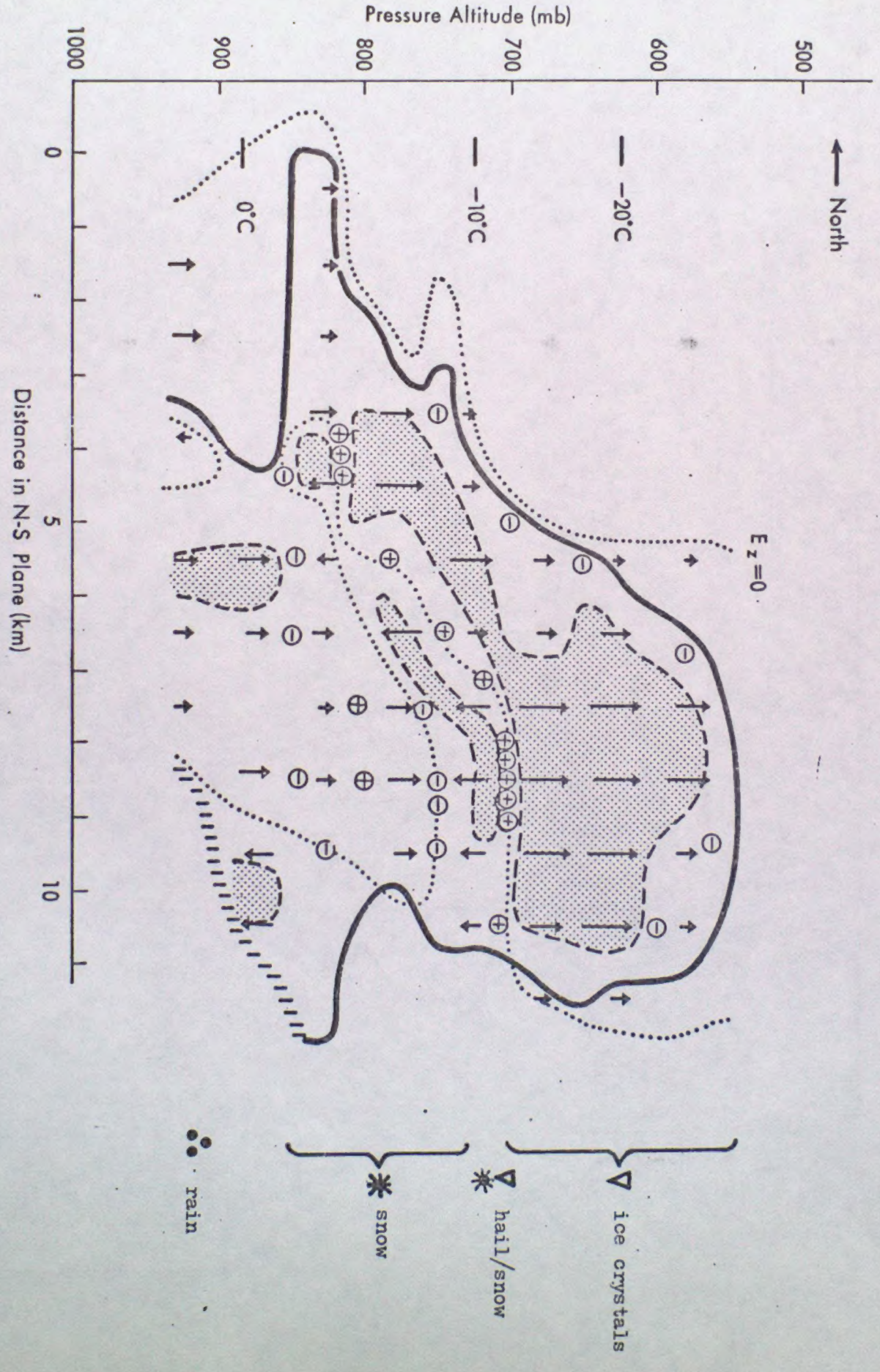
Figure 6 Schematic diagram illustrating the levels and distribution of ground flash charge sources observed in thunderstorms in summertime in Florida and New Mexico and in wintertime in Japan (Krehbiel et al, 1980).

developed through the full horizontal extent of the precipitating region of the storm and appeared to be bounded within its extent. In one instance where cellular structure of the storm was apparent, the strokes selectively discharged regions where the precipitation echo was strongest. The vertical extent of the stroke/charge locations was small in comparison with the vertical extent of the storm. The inter-stroke field changes have been analysed using a point dipole model and found to correspond to predominantly horizontal charge motion that was closely associated with the ground strike sources for the flashes. This activity served effectively to transport negative charge in the direction of the earlier stroke volumes, while subsequent strokes discharged more distant regions of the cloud.

The general conclusion of Khriebel et al, that the negative charge centres are located within a supercooled region of relatively small vertical dimension is consistent with the work of Brook et al (1980). They found that the sources of negative charge for successive strokes of a discharge to ground lie within a relatively limited height range between the -10 and -25°C isotherms, and coincide predominantly with precipitation at these levels. Figure 6 shows a striking result from their studies in Florida, New Mexico and Japan - where convective winter storms were investigated - that the temperatures at which the negative charge centres were located are similar in all three studies, even though the cloud bases are at very different pressure levels.

Bennetts et al (1980) have reported measurements of electric field strength measured in aircraft traverses of convective cloud. The clouds, although electrically active, did not produce lightning. Their data suggest that significant charge generation does not occur before the ice phase is present to a marked degree within the cloud. Nevertheless between two successive traverses through cloud, made in snow at about -8°C some 4 minutes apart, fields of both signs increased by a factor of 4 to about 3kv m^{-1}, suggesting that significant charge separation is possible once ice is present. Figure 7 is a composite of electric field data obtained during 14 traverses of a cloud structure which reached -26°C. It is probable that features separated 'vertically' by more than 100 mb or so bear an evolutionary relationship to one another which is at least as important as that of their apparent spatial association. Nevertheless, provided this is borne in mind, some gross features of the charge distribution can be inferred. Thus between successive penetrations just above and below the -12°C isotherm, the vertical field changed sign on the southern, downshear side of the cloud, suggesting that a positive charge centre was located at this altitude. The measurements are consistent with the presence of a more diffuse negative charge below this region, but no substantial accumulations are indicated. The positive charge centre was located in a region bounded above by almost total glaciation in the form of ice crystals and below by a mixture of snow and hail. It is suggested that the circulation of the cloud, which probably generated at least two interacting cells during its lifetime of over an hour, was insufficient to separate charges and maintain them within the cloud volume; if negative charge resides on precipitation much of it probably fell out of the cloud in that time. Nevertheless charge was being generated in the region of interaction between precipitation size particles and ice crystals; the very strong fields in the upper region of the cloud are consistent with positive charges on the latter.

Figure 7
Distribution of the vertical component of electric field inferred from multiple cloud traverses. Arrows denote the sense and magnitude of E_z. Shaded areas denote $E_z > 32$ Kv m^{-1} (Bennetts et al, 1980)



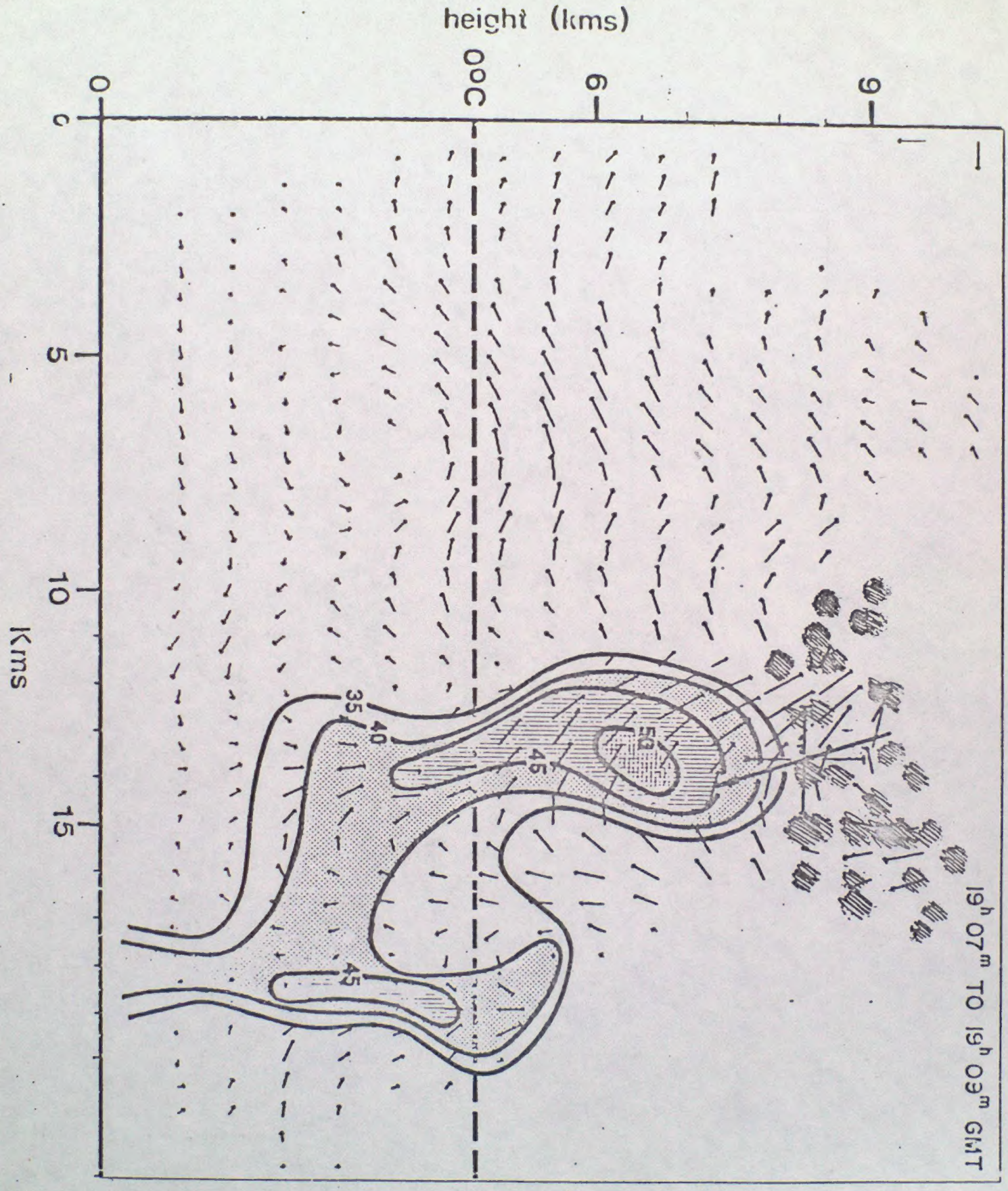


Figure 8 Reflectivity contours and projected Doppler wind field in a vertical plane through a cell at the time of peak electrical activity on 23 August 1978. Hatched areas denote the initial radiation sources from discharges near the analysis plane during 1907 - 1909. (Krehbiel et al, 1980)

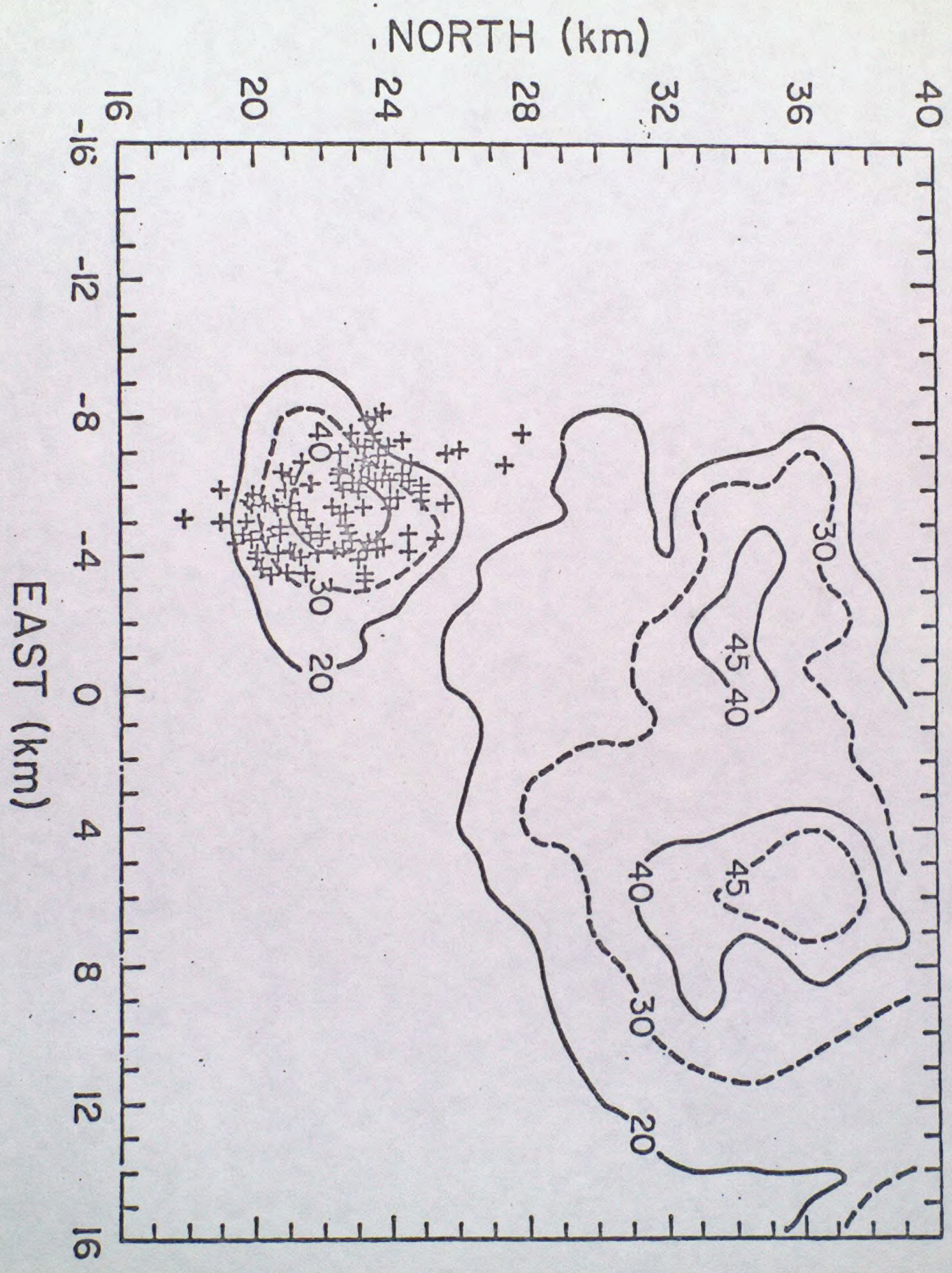


Figure 9 Example of impulse sources (+) for one lightning discharge superimposed upon Doppler radar reflectivity contours (dBZ) from Taylor (1980)

Khriebel et al (1980) found that in one storm system for which Doppler derived wind field data had been obtained, the onset of lightning activity in one of the storm cells followed the development of a strong precipitation echo above the $0^{o}C$ level and coincided with a rapid ($25ms^{-1}$) increase in the speed of the updraught carrying this precipitation. Discharges in the vicinity and perhaps across the region of strong echo gradient were detected - see figure 8.

MacGorman et al (1980) obtained similar, but not identical results by using combined 10 cm radar and VHF/acoustic mapping techniques. It was found that lightning activity was generally located in close proximity to, but not coincident with, the highest reflectivity regions of storms. - see figure 9.

In aircraft and ground based studies of thunderstorms in New Mexico, Gaskell et al (1978) and Christian et al (1980) have concluded that substantial charge densities ($\sim 5nC\ m^{-3}$) are carried on precipitation elements, over horizontal distances of several kilometres in the vicinity of the freezing level. The charges on individual elements were mainly, but not exclusively, negative. They found no discernable relationship between the size of particles and the charges they carried. Lightning activity within a cell was associated with upward motion of precipitation, and was evident where the precipitation rates did not exceed a few millimetres per hour.

The fact that the lightning discharges are strongly correlated with precipitation and that the source regions are found between about -5 and $-25^{o}C$ suggests that ice precipitation is a pre-requisite for significant charging rates. Keuttner (1950), from observations made in thunderstorms capping the Zugspitze in Germany, reported that solid precipitation elements were dominant in the greater part of the thundercloud and were present on 93% of occasions. Snow pellets and pellets of soft hail were the most frequent form of hydrometeor, being present on 75% of occasions, and were always accompanied by strong electric fields, but large hail was relatively rare.

2.3 Quantitative aspects

As was discussed in the lecture on lightning, laboratory experiments have shown that positive corona streamers can be initiated when the local electric field exceeds a critical value of about $6Vcm^{-1}torr^{-1}$, or 3 to $4kvcm^{-1}$ at altitudes relevant to the central regions of thunderstorms. These figures are consistent with the maximum fields of about $4kvcm^{-1}$ which have been recorded around thunderstorms (e.g. Gunn (1948), Winn and Moore (1971)).

Again, as was deduced in the lightning lecture, the N charge in a thunderstorm is typically 40 coulombs or more, some 5 to 10 coulombs being discharged in a stroke; each stroke consisting of several flashes. The charge density in the N region must be about 10 to $20Ckm^{-3}$ or 10 to $20nCm^{-3}$.

3. Theories of thunderstorm electrification

The evidence presented in the preceding sections is strongly suggestive of the role of small hail in carrying the charges that constitute the negative centre of a thunderstorm. In appears likely that the primary charging process or processes involve the collision of small hail with either supercooled droplets or ice crystals. Attention is concentrated on such processes below, but some other mechanisms which appear potentially powerful are considered also.

3.1 Ion Capture Mechanisms

Wilson (1929) pointed out that an electrically polarised hydrometeor in falling through a cloud of ions or charged cloud droplets could, by a process of selective ion capture, acquire a net charge. Thus in a field acting to move positive ions downwards, a hailpellet becomes polarised with a net negative charge on the lower surface.

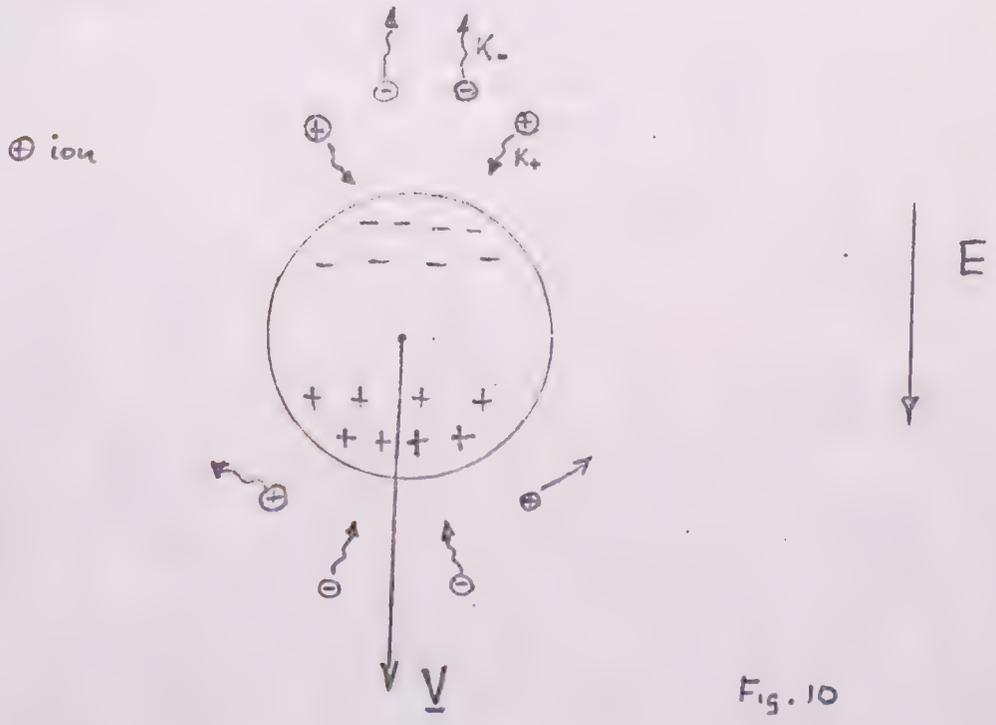


Fig. 10

Negative ions are attracted to it, whilst positive ions are repelled. The rear surface has an equal positive polarisation charge of course, but if the terminal volocity of the particle exceeds the drift velocity of the positive ions, then the latter are not captured. Thus the precipitation particle acquires a net negative charge. The fundamental requirement is that $V > k_+ E$ where V is the terminal velocity, k_+ is the positive ion mobility and E the electric field. V rarely exceeds $8 ms^{-1}$ for hydrometeors other than large hail, and for unattached ions $k \sim 1.5\ cm^{-2}s^{-1}V^{-1}$. Thus the maximum field which can result from this process is $\sim 0.5\ kVcm^{-1}$. Larger fields are possible in principle,

through the capture of slow ions or charged cloud drops. However Wormell has pointed out that the rate of ion production by cosmic rays is only ~ 10 ion pairs $cm^{-3}s^{-1}$ ($\pm$ 6C$km^{-3}hr^{-1}$) so that unrealistic preferential capture efficiencies are required for this source to be a major contributor to thunderstorm electrification. Other secondary sources of ions can be invoked but in general these tend to be productive only in the late stages of electric field growth and/or the processes by which such ions can be transported, at a sufficient rate, to the active part of the cloud, remain obscure. Such criticisms can be levelled at all mechanisms requiring the selective capture of free ions, whether this is due to the Wilson mechanism, the difference between evaporating and condensing drops, Takahashi (1974), the difference in electrochemical potential, Wahlin (1974) or the 'convective' process suggested by Vonnegut (1955) and others.

3.2 Inductive charging of precipitation

The inductive theory, in which a cloud particle (radius, r_a) collides with the underside of a larger precipitation particle (radius, r_b) polarised in an electric field, and seperates charge of magnitude proportional to the field strength (E), is an attractive one because of the inherent positive feedback between charge separation and field strength.

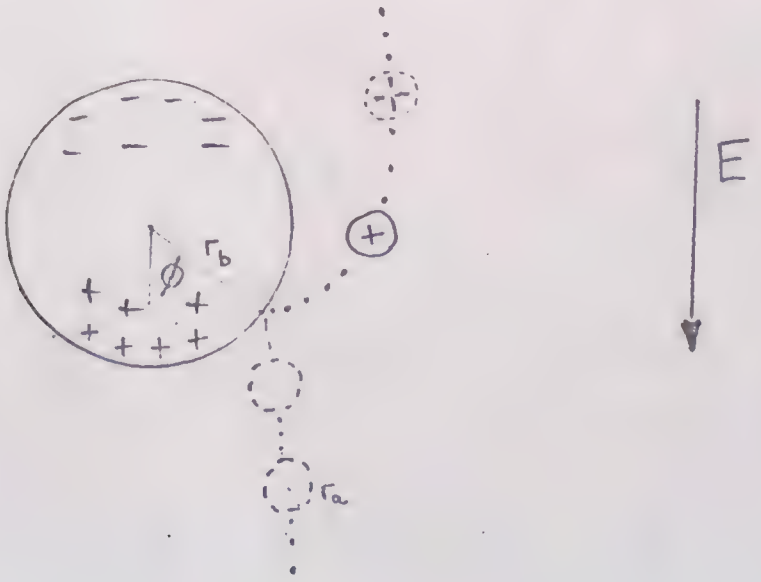


Fig 11

The mechanism has received considerable attention from modellers (for example, Sartor (1961, 1967), Latham and Mason (1962), Scott and Levin (1970) and Mason (1972)), who have concluded that, in principle, water-water, water-ice as well as ice-ice collisions may separate charge at a sufficient rate.

The charge transfer (q) per bouncing interaction, if the large particle has charge Q_B, is:

$$q = \frac{\pi^2}{6}\left(\frac{r_a}{r_b}\right)^2\left[12\pi\varepsilon_0 r_b^2 E\cos\phi + Q_B\right](1 - e^{-t/\tau})$$

where τ is the contact time and ϕ is the angle between the direction of the field and the radius to the point of contact. The maximum achievable charge on the precipitation element is

$$Q_{max} = -12\pi\varepsilon_0 E r_b^2 \cos\phi$$

The water droplet-raindrop experiments of Whelpdale and List (1971) show that separation only occurs when the trajectory of the centre of the droplet passes the drop; otherwise they coalesce. The effective value of cos ϕ allowing bouncing is very small. Furthermore, Jennings (1975) indicates that all collisions result in permanent coalescence for $E > 0.25 kvcm^{-1}$.

Aufdermauer and Johnson (1972) showed that a small fraction ($\sim 0.1\%$) of supercooled droplets, colliding with a hailstone in the presence of an electric field, separate after a collision and transfer charge in accordance with the expression above. They suggested, and Gaskell (1979) and Gaskell and Illingworth (1980) confirmed that these are likely to be glancing collisions, so again cos ϕ is likely to be small. In view of the high concentration of droplets and the fact that for aspherical hail pellets the aerodynamic and electrical 'equators' may be significantly different, it is not possible to discard this mechanism out of hand.

Although separation nearly always occurs following ice-ice collisions, leading to an average $\cos\phi = \frac{2}{3}$, it is not obvious that the charges can flow through the ice during the available time of contact. Gaskell (1979) has pointed out that Herzian theory for a collision between a 100μm ice crystal and hailstone results in a contact time of $\sim 0.3\mu$sec. The bulk resistivity of ice predicts a relaxation time of 10ms and possibly a tenth of this if surface conductivity dominates. Practical evidence supports these ideas. Scott and Levin (1970) found charge transfer compatible with the above equations, using natural snow crystals with slow terminal velocities and long contact times. Latham and Mason (1962) found no field dependent charging for realistic contact times. Buser and Aufdermauer (1974) and Gaskell (1979) observed a much smaller field dependence than implied by the equations. Over the temperature span from -5 to -25°C Gaskell observed no difference in the field dependent charging although the conductivity of ice should change greatly over this range.

The absence of any discernable R^2 relationship between drop charge and size reported by Christian et al (1980) from their aircraft measurements made in New Mexico, has been cited as evidence that the inductive process cannot be of major importance in thunderstorm electrification. Further evidence is required to reinforce this claim.

Rawlins (1980) has carried out numerical modelling experiments, by testing the sensitivity of the inductive process to a number of microphysical assumptions, in a 3d model of cumulonimbus. He finds

that the electric field can reach breakdown threshold within half an hour of the appearance of precipitation, providing that the effect of multiple collisions between ice crystals and hail pellets are neglected; these latter act to discharge earlier charge separation events. As he found it necessary to maximise the charging rate by invoking large numbers of small hail and ice crystals, this assumption may be questionable.

3.3 Non-inductive ice-water interactions

As water freezes on the surface of a hail stone, large potential differences may be developed across the ice-water interface, as a result of the selective incorporation of ions of one sign into the ice lattice. The sign and magnitude of the effect depends on the concentration of the ionic species, and was first studied by Workman and Reynolds (1948,1950). In most cases the solutions tested generated negatively charged ice, so that rapidly growing hailstones, being able to freeze only a fraction of the impinging cloud water, acquire a liquid coat which is shed in the form of small positively charged drops. A similar result is expected to follow from splashing collisions, between hail pellets and large drops.

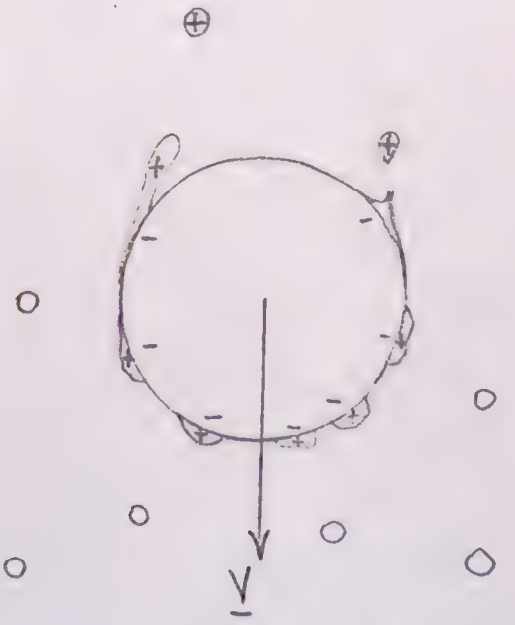
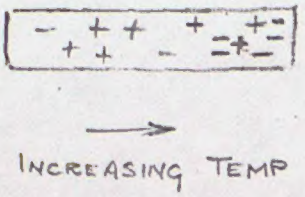


Fig 12

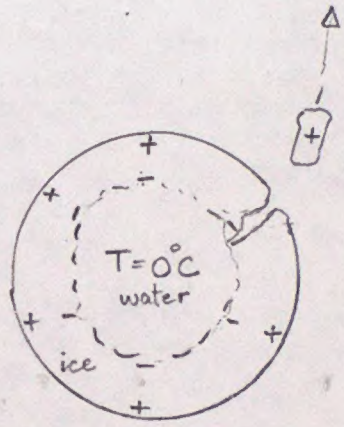
The requirement of large hail, the low probability of splashing events and sensitivity to trace solutes make this an unlikely candidate for a universal charging mechanism. Latham and Warwicker (1980) have shown that the effect is swamped by inductive charging when the ambient field exceeds about $1 vcm^{-1}$. Because in most splashing events water is shed from the rear surface the inductive process is dissipative.

Latham and Mason (1961) reported a powerful process of charge

associated with ice-splinter production during the growth of rime. The process depends on the fact that the concentrations of positive and negative ions in water increase quite rapidly with increasing temperature, and that the hydrogen ion diffuses more rapidly than the hydroxyl ion . Thus the cold end of a piece of ice acquires a greater fraction of positive ions.



(a) Fig. 13



(b) T<0°C

Latham and Mason (1961) suggested that a shell of ice would form around the outside of a supercooled drop soon after nucleation. It was conceived that the inside of the shell being at 0°C and the outside of some lower temperature, would produce positively charged ice splinters as the shell thickened and fractured as freezing proceeded. Unfortunately the mechanism requires unrealistically high rates of splintering to separate sufficient charge.

3.4 Non-inductive ice-ice interactions

Results from laboratory experiments have been variable since the classical studies of Reynolds et al (1957) revealed large charging (hail pellet negative) when ice crystals and supercooled droplets coexisted in the cloud through which a hail pellet moved. Marshall et al (1978) confirmed the efficiency of this charge transfer process when ice crystals rebounded from a rimed target and showed that the results of almost all previous experiments gave, over a crystal diameter range of 10 to 300μm, a value of charge transfer roughly proportional to the square of the ice crystal diameter. In a study of impacts Tabor (1951) notes that the contact area between a plane (or large radius sphere) and a sphere is proportional to the square of the small particle diameter and the impact

velocity. This suggests that charge carriers are actually present on the contact interface and do not flow along the surface - thereby avoiding the time constant problems of the inductive theory. Buser and Aufdermaeur (1974) have demonstrated that such charge transfer is possible in collisions between particles having different surface work functions. Charge transfer takes place to establish the required contact potential. At present there is no time dependent theory for this, but experiments confirm that charge transfer does take place. Illingworth and Latham (1977) and Rawlins (1980) have suggested that a charge transfer of about 10^{-14} to 10^{-13} coulomb (10 to 100fC) is required per collision to provide the necessary charging rate. As the capacitance of a 100 μm sphere close to a plane surface is about 10^{-14} - 10^{-13} farad, the required difference in surface potential is between 0.1 and 1 volt. Takahashi (1973) has reported a difference of 0.2v between the surface potentials of evaporating and condensing ice. Caranti and Illingworth (1980) were unable to reproduce this result but demonstrated a change in surface potential of about 0.4v between ice rimed at -20°C and ice rimed near 0°C or non rimed ice.

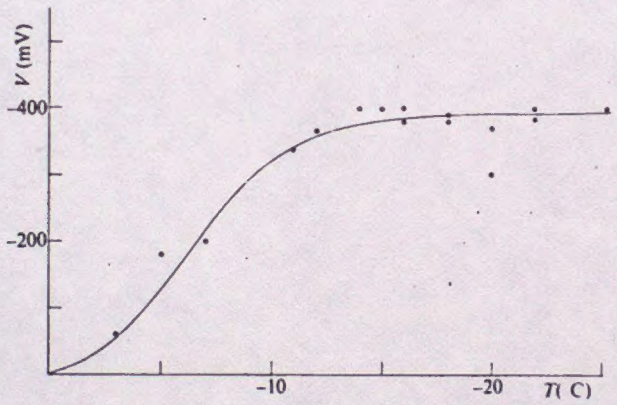


Fig. 14 Change in the surface potential, V, of ice after riming as a function of temperature, T. The unrimed ice surface is assigned a potential of 0V.

This is of the correct sign to produce negatively charged hail pellets when these are riming in the region colder than about -10°C and colliding with unrimed vapour grown crystals. (Recall from lecture 2 that there is a size threshold for the riming of ice crystals.) Such a mechanism fits the observations described in Section 2 very well qualitatively, but more work is necessary to confirm this quantitatively, and to elucidate the physics of charge transfer.

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